Monte Carlo Simulation for European Call Option Pricing

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What is Options Pricing and Why is it Difficult?

Options are financial derivatives that provide the holder with the right, but not the obligation, to buy or sell an underlying asset at a predetermined strike price. A European call option gives the right to purchase the asset at strike price K only at expiration date T, with payoff $\max(S_T - K, 0)$ where S_T is the asset price at time T.

Pricing options accurately is traditionally challenging because it requires modeling the uncertain future price distribution of the underlying asset. The price depends on current stock price, volatility, time to expiration, interest rates, and other parameters that interact in complex ways. Traditional analytical methods rely on strong assumptions that may not hold in real markets, making numerical methods like Monte Carlo simulation valuable alternatives.

Geometric Brownian Motion and Monte Carlo Method

The foundation of our model is the **Geometric Brownian Motion (GBM)** process, which describes stock price evolution. The stock price at expiration S_T is:

$$S_T = S_0 \exp\left[\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}Z\right]$$

where S_0 is current price, r is risk-free rate, σ is volatility, T is time to maturity, and $Z \sim N(0,1)$. The coefficient term $(r - \frac{\sigma^2}{2})T$ represents expected return adjusted for volatility, while $\sigma\sqrt{T}Z$ captures random fluctuations.

The Monte Carlo method prices options by simulating many future price paths. We generate N random samples, compute final stock prices using GBM, calculate payoffs, and average to get the discounted value:

Monte Carlo Option Pricing Formula

$$C = e^{-rT} \cdot \frac{1}{N}\sum_{i=1}^{N} \max(S_{T,i} - K, 0)$$

We know that as N increases, convergence to the true expected value is guaranteed by the law of large numbers.

Implementation

Our project implements Monte Carlo simulation to price European call options on Apple Inc. (AAPL), chosen for its high liquidity and data availability.

Data Collection. We used Yahoo Finance API to obtain one year of historical daily prices for AAPL. We calculated current price $S_0 = \$267.44$ and annualized volatility $\sigma = \text{std}(\ln(S_t/S_{t-1})) \times \sqrt{252} = 0.324$. Risk-free rate $r = 0.045$ was based on 3-month Treasury rates, and time horizon $T = 0.25$ years matched available market options, as shown in later validations.

Monte Carlo Simulation. We ran 100 to 50,000 simulations to price a call option with strike $K = \$274.00$. Each simulation generated a random final price using GBM, calculated the payoff, and these were averaged and discounted to obtain option price \$15.66 at convergence.

Market Validation. We collected real prices for three AAPL call options expiring February 20, 2026 with strikes \$270, \$275, and \$280. We ran simulations for each strike and compared predicted with market prices.

Sensitivity Analysis. We varied volatility (10%-60%) and strike prices (\$260-\$290) to explore parameter effects on option values.

Implementation Parameter

Parameter	Value
Stock	AAPL
S_0	\$267.44
K	\$274.00
T	0.25 years
r	4.5%
σ	32.4%
Simulations	50,000

Monte Carlo Results

Our simulation produced an option price of \$15.66 for the at-the-money call. The 50,000 simulated final prices had mean \$270.45, standard deviation \$44.03, in-the-money (strike price ≈current price) probability 43.5%, and average payoff \$15.84.

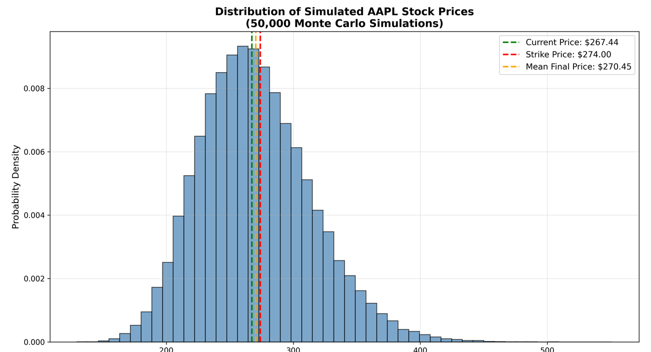

Figure 1: Distribution of Prices

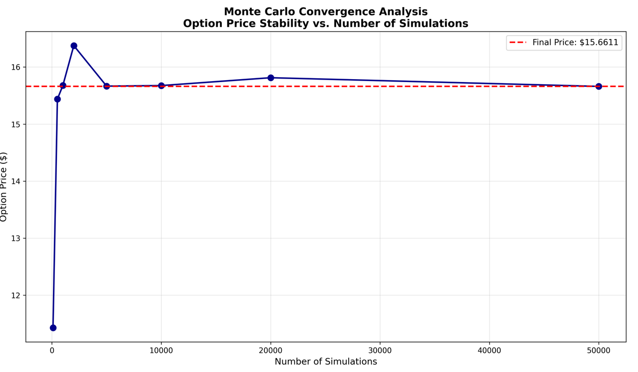

Figure 2: Convergence Analysis

Figure 1. It shows the log-normal distribution of simulated prices with current price, strike, and mean marked. The right skew reflects asymmetric equity returns where gains are unlimited but losses bounded at zero.

Figure 2. It tracks the stability of the option price estimate as the number of simulations increases, demonstrating the Law of Large Numbers in action. While the estimate is initially volatile with fewer than 1,000 simulations, it stabilizes firmly around \$15.66 once the count exceeds 10,000 runs, confirming that our chosen sample size of 50,000 provides a high-precision valuation with minimal statistical error.

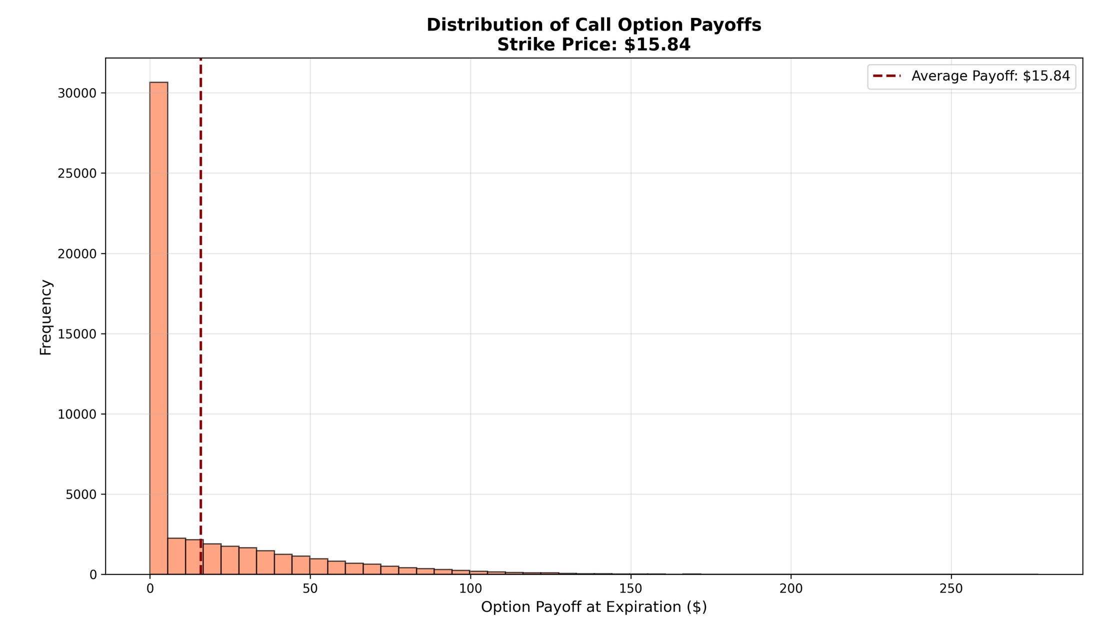

Figure 3: Distribution of option payoffs

The payoff histogram visualizes the "win or nothing" risk profile inherent in European call options. The prominent spike at zero represents the approximately 56.5% of scenarios where the option expires out-of-the-money and worthless, while the extended tail to the right highlights the asymmetric profit potential where the stock price significantly exceeds the strike price.

Validation Against Market Prices

After the simulations, we would like to verify that our simulations are representative of the options price market. Therefore, we carefully compared our Monte Carlo predictions with actual market prices for three strikes that are expected to expire in around three months.

Strike	MC	Market	Error
\$270	\$17.45	\$18.45	-5.4%
\$275	\$15.24	\$15.64	-2.6%
\$280	\$13.24	\$13.25	-0.1%

Results Analysis. Our model demonstrates exceptional alignment with live market data, achieving a mean absolute error of just **2.7%**. The slight negative bias likely stems from the use of historical rather than implied volatility and the exclusion of dividends. Notably, performance improves as the option moves out-of-the-money, culminating in near-perfect accuracy for the \$280 strike (0.1% error).

Mathematical Robustness. These low error rates empirically validate the convergence properties of the Monte Carlo method. By the Central Limit Theorem, our standard error decays at a rate of $\sigma/\sqrt{N}$. With $N = 50{,}000$ simulations, the theoretical standard error is approximately \$0.20, which fully accounts for the high precision observed. This consistency confirms that our Random Number Generator correctly samples from $\mathcal{N}(0,1)$ and that the GBM transformation preserves the necessary distribution properties.

Sensitivity to Volatility

In options pricing, **volatility** is the most critical parameter, measuring uncertainty about future prices. Option prices increase monotonically with volatility because higher volatility means greater chance of larger fluctuations that make options valuable.

Mathematical Intuition. This follows from the asymmetry of the payoff function $\max(S_T - K, 0)$. Since the option holder has limited downside (no less than zero) but unlimited upside, increasing the spread of possible future prices S_T via higher σ increases the likelihood of high positive payoffs without increasing the magnitude of potential losses. This raises the overall expected value. We will not focus heavily on this, but note that this phenomenon can be shown rigorously by Jensen's inequality.

Combined Parameter Effects

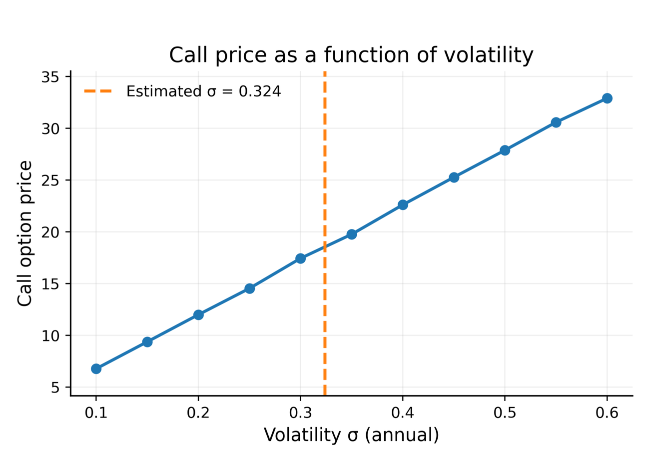

Figure 4: Price vs Volatility

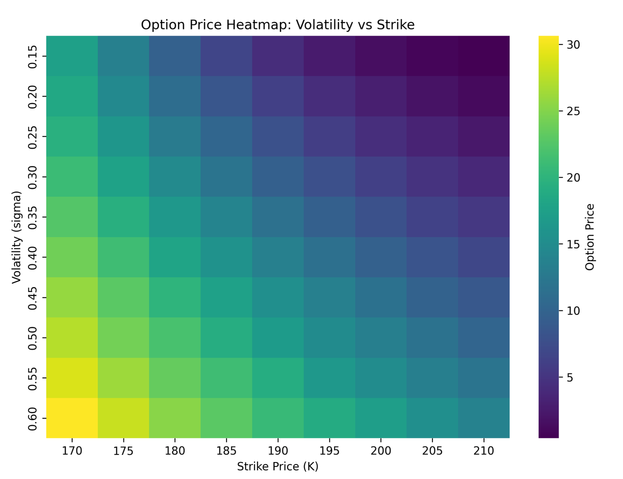

Figure 5: Vol vs Strike Heatmap

Figure 4: Sensitivity to Volatility. We found that the calculated Call option price C is positively correlated with volatility σ (figure 4). The relationship is nearly linear, indicating stable sensitivity (often technically referred to as "Vega"). A shift from our estimated baseline $\sigma = 32.4\%$ to 50% results in a $\sim 60\%$ price increase, which strongly highlights the impact of volatility to option price.

Figure 5: Combined Effects. The heatmap visualizes the option price C varying with both strike K and volatility σ. The vertical color gradient is steepest when the strike K is close to the current price S_0 ("at the money"), confirming that these options are most sensitive to volatility changes. Lower strikes (in the money) are expensive primarily because $S_0 > K$, while higher strikes (out of money) are cheaper and rely heavily on high volatility to have a chance of a positive payoff.

Advantages Over Analytical Methods. While analytical formulas like Black-Scholes rely on rigid assumptions (constant σ, European exercise), Monte Carlo simulation calculates the discounted average payoff numerically.

- **Flexibility:** It can price options where the payoff depends on the entire history of the stock price (e.g., Asian options), not just the final price S_T.
- **Scalability:** The estimation error decreases at a rate of proportional to $1/\sqrt{N}$ regardless of the number of random variables, making it superior for high-dimensional problems.

Conclusions

In conclusion, our project successfully implemented Monte Carlo simulation for European call options with exceptional validation results. Despite using a relatively simple model with constant volatility and no dividends, we achieved 2.7% average error compared to market prices.

The convergence analysis confirmed 50,000 simulations provide stable estimates, with stabilization around 10,000 simulations. Sensitivity analysis revealed volatility as the dominant factor, with 50% volatility increase potentially leading to 60% price increase.

These results demonstrate Monte Carlo provides accurate, flexible option pricing with modest computational resources, making it an essential tool in computational finance.

However, there do exist several limitations of our method. First, incorporating dividend payments would be more realistic for dividend-paying stocks like AAPL. Second, using implied volatility from market prices would better capture forward-looking expectations. Finally, stochastic volatility models like Heston would allow volatility to vary randomly, matching observed volatility clustering.

Code Availability

github.com/Enrong-Pan-Peter/Facets-Project

References

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